Age of High Redshift Objects - a Litmus Test for the Dark Energy Models

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Abstract

The discovery of the quasar, the APM $08279 + 5255$ at $z = 3.91$ whose age is 2-3 Gyr has once again led to "age crisis". The noticeable fact about this object is that it cannot be accommodated in a universe with $\Omega_m = 0.27$, currently accepted value of matter density parameter and $\omega =$ constant. In this work, we explore the concordance of various dark energy parameterizations ($w(z)$ models) with the age estimates of the old high redshift objects. It is alarming to note that the quasar cannot be accommodated in any dark energy model even for $\Omega_m = 0.23$, which corresponds to 1σ deviation below the best fit value provided by WMAP. There is a need to look for alternative cosmologies or some other dark energy parameterizations which allow the existence of the high redshift objects.

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1 Introduction

A lesson given by the history of cosmology is that the concept of the cosmological constant revives in the days of crisis. In the recent past, perhaps the most important reason to reconsider the cosmological constant was to resolve the age crisis in which the universe become younger than its constituents! For example, the discoveries of a 3.5 Gyr old galaxy at z = 1.55 and of a 4.0 Gyr old galaxy at z = 1.43 have been proved to be incompatible in Einstein-de Sitter universe [1, 2, 3].

It was shown that a flat FRW type cosmological model dominated by self-interacting, unclustered fluid with negative pressure collectively known as "dark energy" can accommodate these galaxies and hence alleviate the "age problem" [4]. This idea of dark energy is strongly supported by the observational results of the anisotropy of cosmic microwave background radiation, distance measurements of Type Ia Supernovae and Sloan Digital Sky Survey [5]. One simple candidate of this dark energy is the cosmological constant parametrized by equation of state $\mathrm{p} = \omega\,\rho$ with $\omega = -1$. Although the cold dark matter cosmological constant models are consistent with the present observations, however, there are some fine tuning problems with the unevolving cosmological constant [6].

This has led theorists to explore various dark energy models [7]. However, the nature of dark energy is still unknown. It is not plausible to check every dark energy model by using the observational data. Therefore, model independent probe of dark energy is a good alternative to study its nature. The most common way of measuring dark energy properties has been to parameterize the ω by one or two free parameters and constraining these by fitting the observed data. In the present work we focus our attention to four different popular parameterizations of dark energy. In these models the dark energy components evolves with time. These models are most commonly used in the literature to study the properties of dark energy.

In this paper we discuss observational constraints on various parameterizations of dark energy from age measurements of old high redshift galaxies

(OHRG). These are, namely, the LBDS 53W069, a 4 Gyr-old radio galaxy at z = 1.43 and the LBDS 53W091, a 3.5 Gyr-old radio galaxy at z = 1.55 .

The discovery of a quasar, the APM 08279 + 5255 at z = 3.91 whose age is 2-3 Gyr has once again led to "age crisis" [8]. The noticeable fact about this object is that it cannot be accommodated in a universe with $\Omega_m = 0.27$, currently accepted value of matter density parameter and ω = constant [9]. It is shown that other dark energy scenarios i.e. brane model, generalized Chaplygin gas model etc., are also not compatible with the existence of this object unless we lower the value of H_0 and Ω_m [10]. Therefore in the coming years this object may act as 'litmus test ' for every new dark energy scenario.

The Primary objective of this paper is to use age estimates of the old high redshifts objects to constrain the cosmological models in which dark energy equation of state parameter evolves with time.
The paper is organised as follows: In Section 2 we introduce the various dark energy parameterizations. Cosmic-age redshift is described in the Section 3. Section 4 discusses the results and summarizes our conclusions.

2 $\omega(z)$ Models

We consider spatially flat, homogeneous and isotropic cosmologies. In the presence of non relativistic matter and a dark energy component, the Hubble constant varies with redshift as:

$$\mathrm{H}^2(z) \equiv (\frac{\dot{\mathrm{R}}}{\mathrm{R}})^2 = \mathrm{H}_0^2 \left[\Omega_m \, (\frac{\mathrm{R}_0}{\mathrm{R}})^3 + \Omega_x \, f \right] \tag{1}$$

where the dot represents derivative with respect to time. Further

$$\Omega_m = \frac{8 \, \pi \, \mathrm{G}}{3\mathrm{H}_0^2} \rho_{m0} \quad , \quad \Omega_x = \frac{8 \, \pi \, \mathrm{G}}{3\mathrm{H}_0^2} \rho_{x0}$$

where H_0 is the Hubble constant at the present epoch, while ρ_{m0} and ρ_{x0} are the non relativistic matter density and the dark energy density respectively at the present epoch. Also $f = \rho_x / \rho_{xo}$.

In this paper, we work with the following specific parameterizations for the variation of ω with redshift, i.e.,

$$\mathrm{P0}: \qquad \omega(z) = \omega_o, \tag{2}$$

$$\mathrm{P1}: \qquad \omega(z) = \omega_o + \omega_1 z, \tag{3}$$

$$\mathrm{P2}: \qquad \omega(z) = \omega_o - \omega_2 \ln(1+z), \tag{4}$$

$$\mathrm{P3}: \qquad \omega(z) = \omega_o + \omega_3 \left(\frac{z}{1+z}\right), \tag{5}$$

and

$$\mathrm{P4}: \qquad \omega(z) = \omega_o + \omega_4 \frac{z}{(1+z)^2}. \tag{6}$$

Here ω_o is the current value of the equation-of-state parameter and ω_j ($j = 1, 2, 3, 4$) are free parameters quantifying the time-dependence of the dark energy and these are to be constrained using the observational data [11]. Note that the equation of state of the cosmological constant can be always recovered by taking $\omega_j = 0$ and $\omega_o = -1$.

The model P0 is a simple generalization of dark energy with constant equation of state parameter. This ansatz is referred to as 'quintessence' ($\omega_o \leq 1$) in the literature. Constraints on the Taylor expansion (P1 model) were firstly studied by Cooray & Huterer [12] by using SNe Ia data, gravitational lensing statistics and globular clusters ages. As commented in Ref. [12], P1 is a good approximation for most quintessence models out to redshift of a few and it is exact for models where the equation of state is a constant or changing slowly. P1, however, has serious problems in explaining age estimates of high-z objects since it predicts very small ages at $z \geq 3$ [10]. In reality, P1 blows up at high-redshifts as $e^{3\omega_1 z}$ for values of $\omega_1 > 0$. The empirical fit P2 was introduced by Efstathiou [13] who argued that for a wide class of potentials associated to dynamical scalar field models the evolution of $\omega(z)$ at $z \leq 4$ is well approximated by equation (2). P3 was recently proposed by Linder [14] (see also [15]) aiming at solving undesirable behaviors of P1 at high redshifts. According to [16], this parametrization is a good fit for many

theoretically conceivable scalar field potentials, as well as for small recent deviations from a pure cosmological constant behavior ($\omega = -1$) (see also [17, 18, 19]) for other parameterizations] but this parametrization blows up exponentially in the future as $R \to \infty$ for $\omega_3 > 0$. In P4 parametrization, the dark energy component has the same equation of state at the present epoch (z = 0) and at $z \gg 1$ with rapid variation at low z [20].

Since equations (2-6) represent separately conserved components, it is straightforward to show that the ratio $f = \rho_x/\rho_{xo}$ for (P0)-(P4) evolves, respectively, as

$$\mathrm{P0:} \qquad f_0 = (\frac{R_o}{R})^{3(1+\omega_o)}, \tag{7}$$

$$\mathrm{P1:} \qquad f_1 = (\frac{R_o}{R})^{3(1+\omega_o-\omega_1)} \exp\left[3\omega_1(\frac{\mathrm{R_o}}{\mathrm{R}} - 1)\right], \tag{8}$$

$$\mathrm{P2:} \qquad f_2 = (\frac{R_o}{R})^{3\left[1+\omega_o-\frac{\omega_2}{2}\ln(\frac{\mathrm{R_o}}{\mathrm{R}})\right]}, \tag{9}$$

$$\mathrm{P3:} \qquad f_3 = (\frac{R_o}{R})^{3(1+\omega_o+\omega_3)} \exp\left[3\omega_3(\frac{\mathrm{R}}{\mathrm{R_o}} - 1)\right], \tag{10}$$

and

$$\mathrm{P4:} \qquad f_4 = (\frac{R_o}{R})^{3(1+\omega_o)} \exp\left[\frac{3}{2}\omega_4(\frac{\mathrm{R_o}}{\mathrm{R}} - 1)\right]. \tag{11}$$

Here the subscript o denotes present day quantities and R(t) is the cosmological scale factor. The age of the universe at the redshift z is given by

$$\begin{aligned} \mathrm{H_0}\, t(z) &= \mathrm{H_0} \int_z^\infty \frac{dz'}{(1+z')\mathrm{H}(z')} \\ &= \int_z^\infty \frac{dz'}{(1+z')\sqrt{\Omega_m(1+z')^3 + \Omega_x f}}. \end{aligned} \tag{12}$$

3 Cosmic Age-Redshift Test

In order to constrain the dark energy models under consideration from the age estimate of the above mentioned quasar we follow the line of thought as given in ref. [4]. Firstly, the age of the universe at a given redshift has to be greater than or at least equal to the age of its oldest objects at that redshift.

Hence this test provides lower bound on ω_0 and ω_j. This can be checked if we define the dimensionless ratio:

$$\frac{t(z)}{t_{\rm obj}} = \frac{{\rm H}_0\, t(z)}{{\rm T}_{\rm obj} = {\rm H}_0\, t_{\rm obj}} \geq 1 \ . \tag{13}$$

Here $t_{\rm obj}$ is the age of an old object at a given redshift. For every high redshift object, ${\rm T}_{\rm obj} = {\rm H}_0\, t_{\rm obj}$ is a dimensionless age parameter. The error bar on ${\rm H}_0$ determines the extreme value of ${\rm T}_{\rm obj}$. The lower limit on ${\rm H}_0$ is updated to nearly 10% of accuracy by Freedman [21]: ${\rm H}_0 = 72 \pm 8$ km/sec/Mpc. We use minimal value of the Hubble constant, ${\rm H}_0 = 64$ km/sec/Mpc, to get strong conservative limit.

Komossa and Hasinger (2002) [22] have estimated the age of the quasar APM $08279 + 5255$ at redshift z= 3.91 to be between the interval 2-3 Gyr. They use an Fe/O =3 abundance ratio (normalised to the solar value) as inferred from X-ray observations to draw the conclusion. An age of 3 Gyr is concluded from the temporal evolution of Fe/O ratio in the giant elliptical model (M4a) of Hamman and Ferland [23]. The age estimate of 2 Gyr is inferred by using the "extreme model" M6a of Hamman and Ferland for which the Fe/O evolution is faster and Fe/O = 3 is reached after 2 Gyr. Friaca, Alcaniz and Lima (2005) [10] reevaluate the age for APM 0879+5255 by using a chemodynamical model for the evolution of spheroids. They quote the age of this old quasar at z = 3.91 as 2.1 Gyr. To assure the robustness of our analysis we use the lower age estimate for our calculations. The 2 Gyr old quasar at z = 3.91 gives ${\rm T}_{\rm q} = 2.0{\rm H}_0$ Gyr and hence $0.131 \leq {\rm T}_{\rm q} \leq 0.163$. It thus follows that ${\rm T}_{\rm q} \geq 0.131$.

Similarly, for LBDS 53W069, a 4 Gyr-old radio galaxy at z = 1.43, we have ${\rm T}_{\rm g} \geq 0.261$ and for the LBDS 53W091, a 3.5 Gyr-old radio galaxy at z = 1.55, ${\rm T}_{\rm g} \geq 0.229$.

4 Results and Discussions

It is now a well accepted fact that the universe is accelerating and is dominated by a smoothly distributed cosmic fluid, referred to as 'dark energy'. A number of observational tests have been proposed in the literature to study the nature of dark energy, including SNe Type Ia luminosity distances, grav-

itational lensing, Cosmic Microwave Background anisotropy, angular size-redshift relationship of compact radio sources, Lyman-alpha forest etc. Essentially most of the proposed observational tests are based on the measurement of distance-redshift relationship. The age estimate of old high redshift objects provides an independent way of studying various dark energy models. As opposed to the other tests listed above, this method is based on the time-dependent observables. In this work, we explore the concordance of various dark energy parameterizations with the age estimates of the old high redshift objects.

Fig. 1 summarizes the results for the P0 model. The model is a simple generalization of dark energy with constant equation of state parameter. This parametrization fails to accommodate the old quasar for $\Omega_m = 0.27$ as shown in Fig. 1 . This is a serious problem. The two old galaxies put a upper bound on ω_0 as: $\omega_0 \leq -0.32$ for LBDS 53W069, a 4 Gyr-old radio galaxy at z $= 1.43$ to exist and $\omega_0 \leq -0.24$ for the LBDS 53W091, a 3.5 Gyr-old radio galaxy at z $= 1.55$ to exist. In case the dark energy is modeled as cosmological constant ($\omega_0 = -1$), the old quasar is accommodated only if $\Omega_m < 0.21$.

The results for P1 model are shown in Fig. 2. For a given old galaxy, the line represents the minimal value of its age parameter ($\mathrm{T_g} = \mathrm{H_0} t_\mathrm{g}$). Of the two galaxies, the radio galaxy, LBDS 53W069, at z $= 1.43$ provides tighter constraints. For this galaxy, $\mathrm{T_g} \geq 0.261$. The age estimates provide the constraints: $\omega_0 \leq -0.31$ and $\omega_1 \leq 0.96$ for $\Omega_m = 0.27$. The constraints change to $\omega_0 \leq -0.28$ and $\omega_1 \leq 1.08$ for $\Omega_m = 0.23$. Here, we have worked with the following ranges: $-2 \leq \omega_0 \leq 0$ and $0 \leq \omega_1 \leq 4.0$. We once again find that the parameterization fails to accommodate the old quasar for $\Omega_m = 0.27$. It fails to do so even for $\Omega_m = 0.23$.

In Fig. 3, we display the results for P2 model. For this model, we work with the following ranges: $-2 \leq \omega_0 \leq 0$ and $-2 \leq \omega_2 \leq 0$. The radio galaxy LBDS 53W069 puts the constraints: $\omega_0 \leq -0.31$ and $\omega_2 \geq -1.95$ for $\Omega_m = 0.27$. The constraints change to $\omega_0 \leq 0. - 0.28$ and $\omega_2 \geq -1.99$ for $\Omega_m = 0.23$. The parameterization fails to accommodate the quasar even for $\Omega_m = 0.23$.

In Fig. 4 we show the parametric space $\omega_0 - \omega_3$ for the P3 model. We work with the ranges: $-2 \leq \omega_0 \leq 0$ and $0 \leq \omega_2 \leq 4.0$. The tighter constraint is given by 4.0-Gyr galaxy(53W069) at z = 1.43 as expected. The constraints are: $\omega_0 \leq -0.31$ and $\omega_3 \leq 3.29$ for $\Omega_m = 0.27$. The constraints change to $\omega_0 \leq 0. - 0.28$ and $\omega_3 \leq 3.39$ for $\Omega_m = 0.23$.

Fig. 5 shows the parametric space $\omega_0 - \omega_4$ for P4 model. The ranges for the parameters are: $-2 \leq \omega_0 \leq 0$ and $0 \leq \omega_4 \leq 4.0$. For $\Omega_m = 0.27$, we get the constraint $\omega_0 \leq -0.31$ while the entire range of ω_4 is allowed. For $\Omega_m = 0.23$, we get $\omega_0 \leq -0.28$.

It is interesting to note the following points:
1. If we concentrate on the two galaxies, for all the models considered above, galaxy data rules out non-accelerating models of the universe. The main issue whether the dark energy is a cosmological constant or if it is evolving with redshift, cannot be resolved with these data points. Since $\omega_0 < -0.31$ and wide range of ω_j (j = 1,2,3,4) is allowed for $\Omega_m = 0.27$. Therefore, with these data points the age-redshift test does not rule out cosmological constant as a dark energy candidate.

2. *The old quasar cannot be accommodated in any of the dark energy parameterizations* for any range of w_0 and w_j even for $\Omega_m = 0.23$ which corresponds to 1σ deviation below the best fit value provided by WMAP.

The quasar can be accomodated if the values of H_0 or/and Ω_m are further lowered down from their currently accepted values. However, reduction of the matter contributions would disturb the galaxy formation.

Since almost all the age of the universe is at low redshift (z = 0-2), the galaxies may have formed nearly at the same epoch, regardless of their constraints on the redshift space. The effect of dark energy on the galaxy formation epoch has been studied by Alcaniz and Lima (2001)[24]. A large matter contribution results in a larger value of z_f. It is shown that lower the value of w, lower the value of redshift of formation of galaxies, z_f. For example, the galaxy 53W091 with $\Omega_m = 0.3$, $w = -1/3$ gives $z_f \geq 20.3$. The $z_f \geq 5.2$ for the same galaxy with $w = -1$. P. J. McCarthy et al. (2004) report a

$z_f = 2.4$ from conservative age estimates for 20 colour selected red galaxies with $1.3 < z < 2.2$ [25]. This may require $w < -1$ with $\Omega_m \sim 0.27$.

Recently, this old quasar APM $08279 + 5255$ is studied in detail in reference with various other dark energy scenarios [9, 10]. Friaca, Alcaniz and Lima also work with the P0, P1 and P3 parameterizations using the old quasar [10]. They, however, consider $w_o, w_1$and w_3 as fixed parameters. They report that this object is not compatible with any model unless the values of H_0 or/and Ω_m are further lowered down from their current accepted values.

As discussed in Section 3, the estimated age of the quasar APM $08279+5255$ lies between 2-3 Gyr. In this paper, we use the lower age estimate of 2 Gyr for the quasar in order to increase the robustness of our result.

It looks like we are once again in a situation of crisis. There is a need to look for alternative cosmologies or some other dark energy parameterizations which allow the existence of the high redshift objects. Linear coasting cosmology [26] and Brane world models [27] accommodates this old quasar comfortably. These models are also concordant with the host of other cosmological observations [27, 28].

In this paper we use the method based on the absolute age determination of the high redshift objects. A quite similar approach is also used by Capozziello et al. [29]. They use the look back time to high redshift objects (clusters of galaxies) rather than their cosmic age to constrain various dark energy models. They, however, point out that galaxy clusters are not good candidates for such a study because it is difficult to detect a enough number of member galaxies at redshift greater than 1.3. Similarly, cosmic age from globular clusters is used as a tool to constrain dark energy parameterizations [30]. The results from this method show that age limits plays significant role in understanding the properties of dark energy.

Jimenez and Loeb propose the use of relative ages as a tool for constraining the cosmological parameters [31]. They emphasize that the relative age is better determined than the absolute age because systematic effects on the absolute scale are factored out(for a fractional age difference $<< 1$). The

statistical significance of the results depends upon the samples of passively evolving galaxies with high-quality spectroscopy. All samples need to have similar metallicities and low star formation rates [32]. But the small number of galaxies in the sample again donot give enough accuracy to the constrain $w(z)$.

The calibration of the absolute age of high redshift objects is subject to observational (precise measurements of their distances, turn off luminosity) and theoretical (all the aspects of stellar astronomy) uncertainties [33]. Never-the-less it is an independent elementary means to test cosmological models. In future, with more input data, this simple tool may turn into a powerful one.

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Model	Constraints from the Galaxies	Constraints from the Quasar
P0	$\omega_0 \leq -0.32$	Not Accommodated
P1	$\omega_0 \leq -0.31$, $\omega_1 \leq 0.96$	Not accommodated
P2	$\omega_0 \leq -0.31$, $\omega_2 \geq -1.95$	Not accommodated
P3	$\omega_0 \leq -0.31$, $\omega_3 \leq 3.29$	Not accommodated
P4	$\omega_0 \leq -0.31$, entire range of ω_4 is allowed	Not accommodated

Table 1: The constraints on ω_0 and ω_j are with $\Omega_m = 0.27$ and $\mathrm{H}_0 = 64$ km/sec/Mpc.

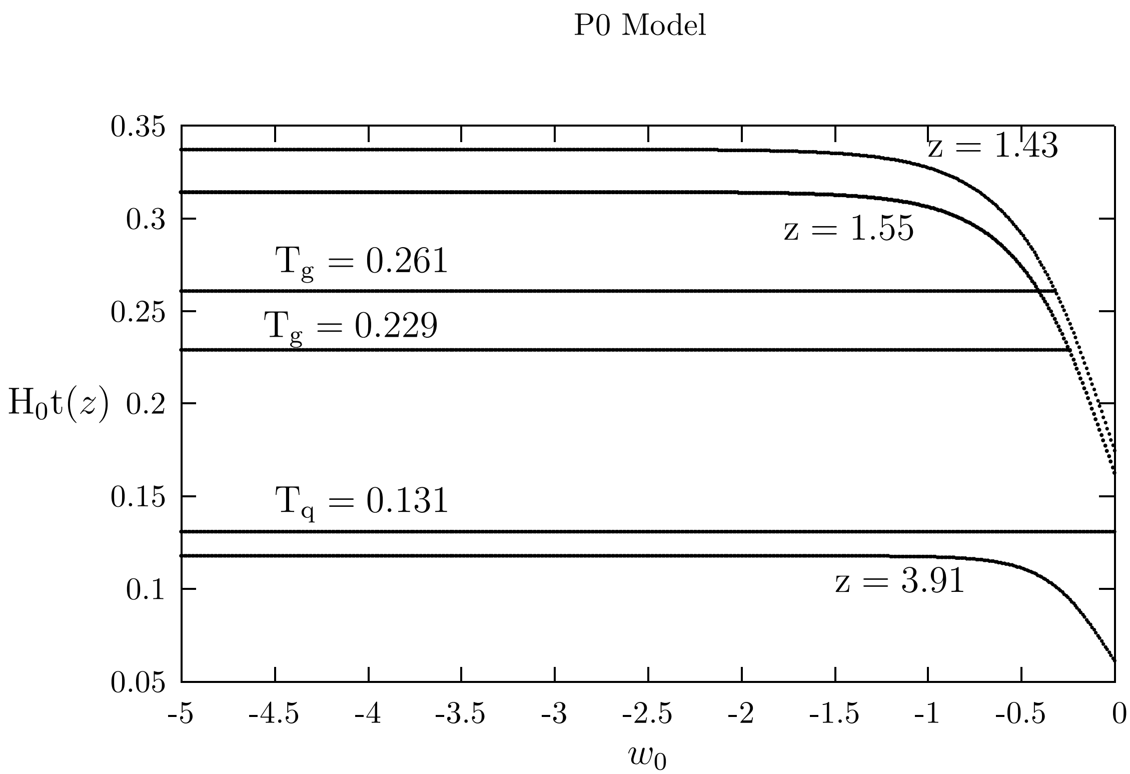

Figure 1: $H_0 t(z)$ as a function of ω_0.

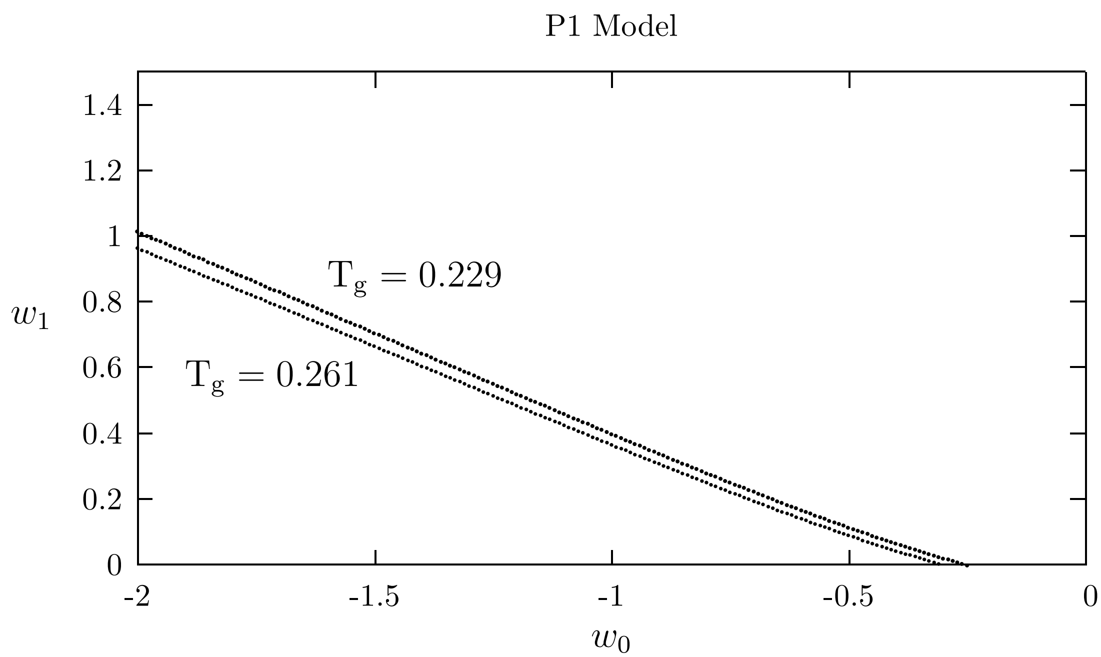

Figure 2: The two contours in the parametric space correspond to the constant values of $\mathrm{H_0 t_{obj}}$ for the two galaxies.The space below the curve $\mathrm{T_g} = 0.261$ is the allowed region.

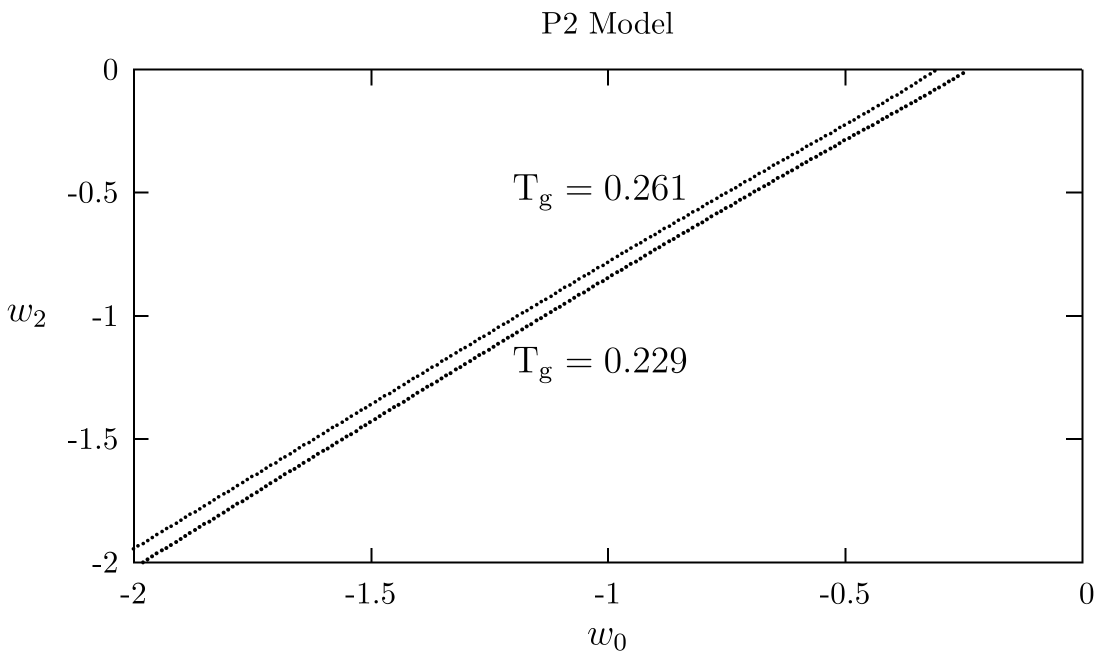

Figure 3: The space above the curve $T_g = 0.261$ gives the possible combinations of (ω_0, ω_2) which allow both the old high redshift galaxies to exist.

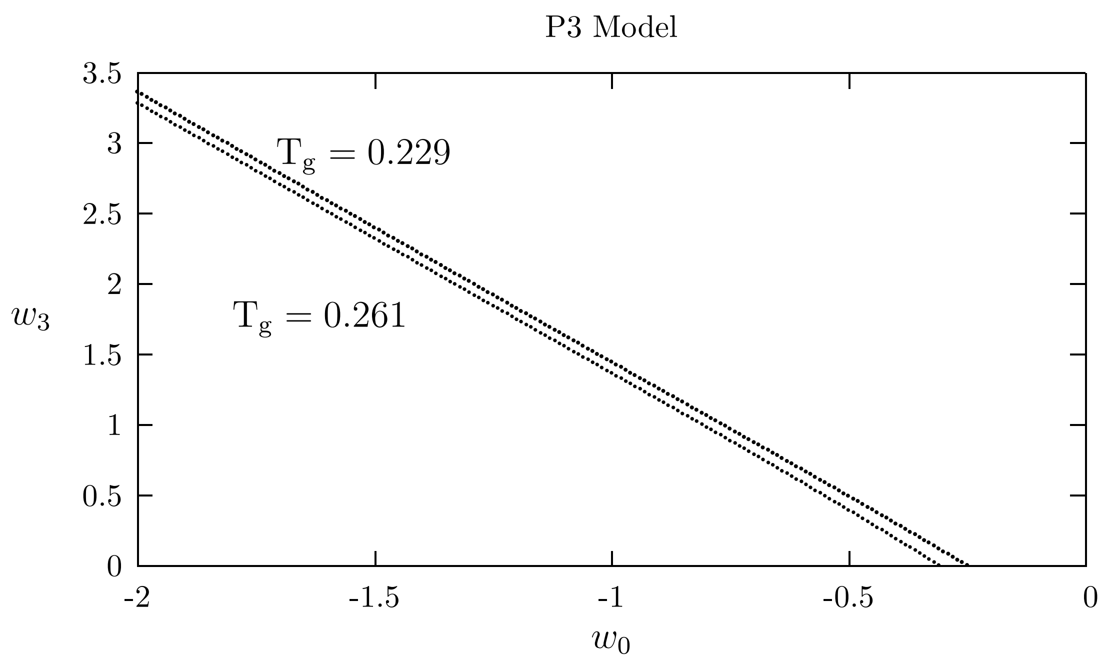

Figure 4: The space below the curve $T_g = 0.261$ gives the possible combinations of (ω_0, ω_3) which allow both the old high redshift galaxies to exist.

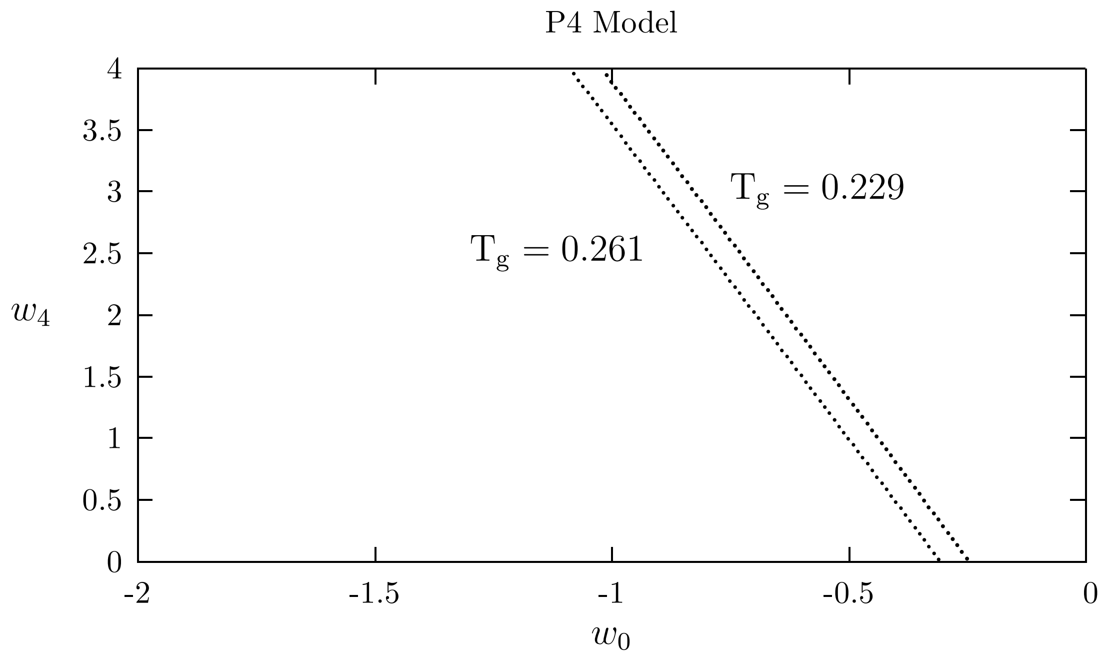

Figure 5: The region left to the $\mathrm{T_g} = 0.261$ gives the possible combinations (ω_0, ω_4) for which both the galaxies can exist.